Appendix A

Exercises and Solutions

Exercises Chapter 1

1. [1] What is the probability of rolling a one on an eight-sided die?
2. [1] What is the probability of rolling an even number on a normal die?
3. [2] How many combinations can hit a single blot from two points, with distance 7 and 8?
4. [2] What is the probability of coming in with two checkers from the bar on a two point board?

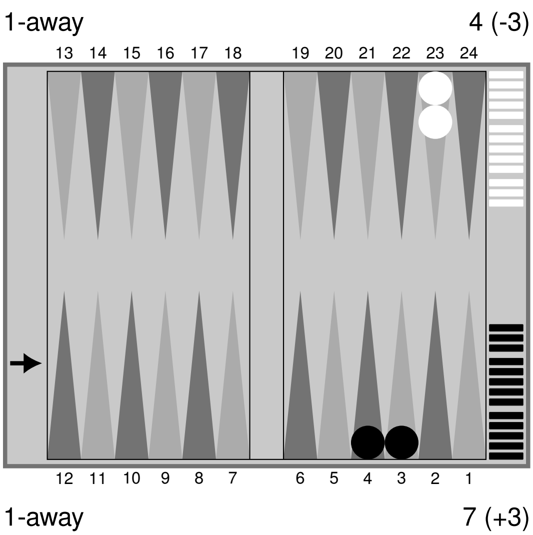

Diagram A.1: What is the probability for Black to win?

5. [2] Calculate Black's probability of losing in position 1.4 under the assumption that he mistakenly plays 5/2 if he rolls a 21.
6. [3] What is the probability for Black to win in diagram B.1?

Solutions Chapter 1

1. For an eight sided perfect die the probability of rolling each number is equal. Since the sum of all probabilities has to be 1, we know that $1 = 8 \cdot p$, where p is the probability we are interested in. Solving this equation gives $p = \frac{1}{8}$. Thus, the probability of rolling a 1 is $\frac{1}{8} = 0.125\%$
2. There are three even numbers on a die. Each occurs with a probability of $\frac{1}{6}$. Thus the total probability is $3 \cdot \frac{1}{6} = \frac{3}{6} = \frac{1}{2} = 50\%$
3. For distance seven there are six combinations that hit: ⚅⚀, ⚀⚅, ⚄⚁, ⚁⚄, ⚃⚂, ⚂⚃. For distance eight, there are a different six combinations (refer to figure 1.6 on page 9). All in all that makes 12 combinations, or a probability of $\frac{12}{36} = \frac{1}{3}$. Note that this is more likely than hitting a one shot, for which are 11 rolls.
4. Since there are four open points on a two point board, both dice must roll one of these four numbers. $4^2 = 16$ gives us 16 combinations, or a probability of $\frac{16}{36} = \frac{4}{9}$.
5. To calculate this we can take the formula we developed at the end of section 1.5, and change the last fraction from $\frac{2}{36}$ to $\frac{10}{36}$. This was the probability of rolling the second 21, but now every ace except 11 will also lose:

$$\frac{17}{36} \cdot \frac{6}{36} + \frac{2}{36} \cdot \frac{30}{36} \cdot \frac{10}{36} \approx 9.16\%$$

   Since Black loses only 8.1% of the time after the best move, misplaying the 21 costs him more than one percent winning chances.
6. Let us again first calculate the probability of Black losing. To lose, Black must roll an ace or a deuce. According to figure 1.8 on page 11, there are 20 combinations. ⚁⚁ will win though, so there are 19 combinations left. Now White has to roll one of his winning numbers. There are 11 combinations which involve aces, but double ace will win anyway, so there are 10 combinations left in which White will not win. Thus, in $36 - 10 = 26$ combinations White will win and Black will lose.

Black can also lose by rolling 21 twice. For this to happen, White must roll one of his 10 bad numbers in between, otherwise Black won't roll a second time. Putting this together:

$$\frac{19}{36} \cdot \frac{26}{36} + \frac{2}{36} \cdot \frac{10}{36} \cdot \frac{2}{36} \approx 38.2\%$$

We asked for the probability that Black will win, but we calculated instead the probability that White will win. Since the sum of both players' winning probability is always 100%, we can derive the final result: $100\% - 38.2\% = 61.8\%$.

Exercises Chapter 2

1. [1] What is the equity of a player in a DMP position if his probability of winning is 35%?
2. [1] What is the winning probability of a player in a DMP position if his equity is 0.4?
3. [2] What is the equity of a player who wins 55% of all games, among them 17% gammons? His opponent wins 9% gammons.
4. [2] What is the equity of a player who wins 28% of all games, among them 10% gammons and 1% backgammons? His opponent wins 38% gammons and 3% backgammons.

Solutions Chapter 2

1. Using definition 2.1 on page 18: $2 \cdot 35\% - 1 = 2 \cdot 0.35 - 1 = -0.3$
2. Using corollary 2.1 on page 18: $\frac{0.4+1}{2} = 0.7 = 70\%$
3. Using definition 2.3 on page 20: $0.55 + 0.17 - (0.45 + 0.09) = 0.18$

   Note that the opponent wins 45% of the games, hence the 0.45 in the formula.
4. If the player wins 28% of the games, the opponent wins 72%. We put this together and use definition 2.3:

$$(0.28 + 0.1 + 0.01) - (0.72 - 0.38 - 0.03) = -0.74$$

Exercises Chapter 4 1. [3] List all market losing sequences in the following endgame position: Black has a checker on the 6 point and one on the 4. White has one checker on the 6 point and one on the 3.

2. [3] Calculate the volatility after double/take in position 3.2 (one checker on the 6 point versus one checker on the ace).

Solutions Chapter 4

1. - All of Black's rolls that take both checkers off are market losers, namely 65, 64, 33, 44, 55, 66 (8 numbers).
   - After 31, 11, and 21 Black doesn't lose his market no matter what White rolls.
   - After the other 23 rolls Black loses his market when White does not take both checkers off, which is the case after 12, 13, 14, 15, 16, 23, 24, 25, 26, 34, 35, 45, 11, 22 (26 numbers).
2. The equity after double/take E_{DT} is 1.0 in this position. When White wins her equity is 2, so the difference is $2-1=1$. When White loses, her equity is -2 and the difference between losing and the current equity is $-2-1=-3$. White will lose after 9 rolls, so there are all together $9 \cdot 36$ losing sequences. After the other $27 \cdot 36$ sequences she will win the game. According to definition 4.3 the volatility is:

$$\frac{27 \cdot 36 \cdot 1^2 + 9 \cdot 36 \cdot (-3)^2}{1295} \approx 3.00$$

Exercises Chapter 5

1. [1] You are behind 7-away/2-away. What is the fair price you would offer to your opponent if you wanted to stop now?
2. [2] Calculate the EMG equity at the beginning of a game at 3-away/6-away.
3. [4] How much in terms of match winning chances does it cost to make a 150 millipoints error at 7-away/4-away with the cube on 2? Conversely, what is the size in EMG equity of an error at 7-away/4-away with the cube on 2 that costs 1% in MWC?

Solutions Chapter 5

1. According to figure 5.1 the match equity at 7-away/2-away is 16%:

$$2 \cdot p - 1 = 2 \cdot 16\% - 1 = 2 \cdot 0.16 - 1 = 0.32 - 1 = -0.68$$

You should pay 0.68 units to your opponent.

2. At 3-away/6-away the match equity is 71%. After winning a point it would be 80%, and after losing a point it would be 65%. According to definition 5.2 the EMG equity is thus:

$$E = 2 \cdot \frac{M - M_l}{M_w - M_l} - 1 = 2 \cdot \frac{71 - 65}{80 - 65} - 1 = 2 \cdot \frac{6}{15} - 1 = -0.2$$

   This intuitively means that, on average, gammons and the cube will benefit the 6-away player more.

3. If we win, the score will be 5-away/4-away, and if we lose it will be 7-away/2-away. According to the match equity table in figure 5.1 M_w is thus 42, and M_l is 16. The error in normalized equity is 0.150, which is $E_1 - E_2$ in the formula we derived in section 5.3.5:

$$\left(E_1 - E_2\right) \frac{(M_w - M_l)}{2} = 0.150 \cdot \frac{42 - 16}{2} \approx 1.95$$

   With the precision that we used in figure 5.6 for the match equities the exact result would be 1.987%. This is a little bit more than twice as high as with the cube on 1 (0.89%).

   For the reverse calculation we did not derive a formula in the text. We can do this in a similar way, starting with definition 5.2 and subtracting two match winning chances M_1 and M_2.

$$\begin{aligned} & 2 \cdot \frac{M_1 - M_l}{M_w - M_l} - 1 - \left(2 \cdot \frac{M_2 - M_l}{M_w - M_l} - 1\right) \\ = \; & \frac{2}{M_w - M_l} \left((M_1 - M_l) - (M_2 - M_l)\right) \\ = \; & \frac{2}{M_w - M_l} (M_1 - M_2) \end{aligned}$$

   We are supposed to calculate the corresponding EMG equity for a mistake that costs 1% in MWC. Thus we have $M_1 - M_2 = 1\%$. Since the cube is on 2, we again have $M_w = 42\%$ and $M_l = 16\%$. We get:

$$\frac{2}{M_w - M_l} (M_1 - M_2) = \frac{2}{42 - 16} \cdot 1 \approx 0.077$$

   Using the exact match equities the corresponding error in EMG equity is about 75.5 and not 77 like in our calculation. The size of the mistake is a little less than half the size of the error with the cube on 1 (168).

Exercises Chapter 6

1. [2] Calculate the probability of winning the match after taking the cube in position 6.1 at 6-away/4-away. Is it better to take or to pass?
2. [1] Calculate the match equity according to Neil's Numbers for the score 10-away/7-away

Solutions Chapter 6

1. If we take and lose, we will be at 6-away/2-away for 20% match equity. If we take and win, we will be at 4-away/4-away for 50%. Considering that we win with probability $\frac{9}{36}$, we get:

$$M = 50 \cdot \frac{9}{36} + 20 \cdot \frac{27}{36} = 27.5.$$

   Passing brings us to 6-away/3-away for 29% match equity. This is higher than 27.5%, and thus it is better to pass.

2. Neil's number for 10-away is $5\frac{1}{3}$. The difference between 10 and 7 is 3. We have to multiply the two numbers: $3 \cdot 5\frac{1}{3} = 16$. We are trailing, so we have to subtract the 16 from 50%, which is 34%. According to the MET we use the correct value is 34.37%.

Exercise Chapter 7

1. [2] Calculate from the match equity table the double point for the recube to 4 at 5-away/6-away.

Solution Chapter 7

1. After winning with the cube on 2 our match equity will be 71%, and after losing it will be 42%. With the cube on 4 our match equity would be 89% after a win and 26% after a loss. According to definition 7.1 the risk is $42 - 26 = 16$ and the gain is $89 - 71 = 18$. The double point is:

$$\frac{r}{r+g} = \frac{16}{16+18} = \frac{16}{34} \approx 47\%$$

   Note that this differs from the 49% shown in figure 7.2(b) due to rounding mistakes in the match equity table.