

The Theory of Backgammon

Mathematics, Logic, and Intuition

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Introduction

Despite its simple rules, backgammon is a complicated game. As a consequence, players' skill levels can vary widely, and the difference between a world class player and an intermediate is enormous. This means that, in a book about the theory of backgammon and its applications, there are necessarily parts that are not so very interesting for the former and others that are too advanced for the latter. My goal was to strike a balance between the two extremes, so that everybody can benefit from some parts of this book.

The book is divided into three main parts. Many of the more intriguing topics discussed in part II build upon content introduced in part I, the foundation section. I would therefore recommend that the reader fully understand the basic theory in part I before continuing on to part II. In my opinion part I, provides helpful information even for experienced players with ample theoretical knowledge. Part III, which deals with the dynamics which arise from non-optimal play, is somewhat independent, although a basic understanding of the previously discussed concepts is nonetheless required.

In addition to the big skill differences within the player pool, there are also a wide variety of ways in which a player may choose to approach the game. Whereas some players are rather math-based and do many calculations over the board, others use a much more intuitive approach and rely mainly on feeling and pattern recognition. Since the randomness of the dice and the calculus of the doubling cube are an essential part of the game, some basic math is indispensable to describing the theory of backgammon. Personally I like the mathematical aspects of backgammon, but I am aware that my preference is not shared by everybody. Therefore, I have marked some paragraphs with arrows. The content between  and  is mathematical in nature and not strictly necessary to understand the rest of the book if you simply take the results for granted. I also clearly note when an entire section is of a rather technical na-

ture, and its content is not necessary for understanding the rest of the text.

As far as it is possible, the backgammon theory developed in the book proceeds on two parallel tracks, one that is formal and mathematical and another that relies more upon intuition and basic logic. The analysis of sample positions using the software eXtreme Gammon plays an essential role in illustrating the concepts explained. I used XG's standard parameters for rollouts if not explicitly mentioned otherwise. It is important to note, however, that the results of computer analysis have to be taken with a grain of salt, except when an exact calculation is possible as it is, for instance, in some endgame positions. No computer program plays perfectly, and it is therefore impossible to *know for certain* whether the rollout result of a position is correct. Nevertheless, I will pretend throughout the book that rollouts are correct for the sake of simplicity, and also because we don't have any better method available. Therefore, a statement like "the rollouts indicates that decision A is a mistake of size 0.063" must not be taken literally. The example positions are chosen in such a way that we can be fairly confident that the rollout results come close to the truth. However, we simply have to accept that the "true" nature of most positions cannot be known without solving backgammon.

The position diagrams are more or less self-explanatory. The numbers on the right side of a standard position diagram are the numbers that you can find in XG's analysis window. When I refer to a position in the text, I use various terms that all mean the same thing. For instance, "the position in diagram 1.1", "diagram 1.1", and "position 1.1" all refer to the position shown in the diagram labeled 1.1. Diagrams that don't show a backgammon position are labeled as figures and they are enumerated separately.

The book is not written for beginners, and I assume that the reader is familiar with the most common backgammon terms like, for instance, advanced anchor or Crawford game. The most important terms for developing the theory, however, are all formally defined. Note that throughout the book I use these terms strictly in the sense of my formal definitions to avoid any confusion with slightly different definitions that you might find elsewhere.

This second edition of *The Theory of Backgammon* differs slightly from the first one. There are some minor improvements, such as the correction of typos, the simplification of some formulas, and better explanation of some tables. However, in a few chapters there are significant changes: In chapter 2 I added a section about cubeful equities for checker play decisions, as well as a chapter on this topic in the appendix. In chapter 8 I rewrote the part about the "free pass" and added a position. In chapter 13 I removed one position in exchange for two new positions that I found far more instructive. And I rewrote some parts of chapter 17 to make the guidelines for cube strategy in a 5 point match easier to understand.